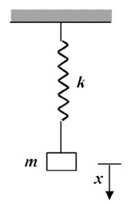
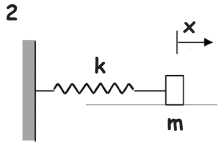
OSCILLATIONS 01

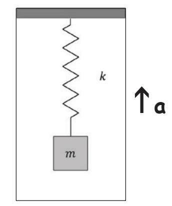
TIME PERIOD OF S.H.M

1


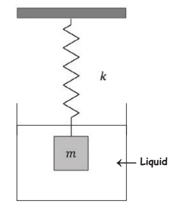
F=-kx
$T=2\pi\sqrt{\frac{m}{k}}$

2


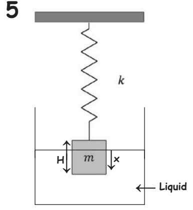
F=-kx
$T=2\pi\sqrt{\frac{m}{k}}$

3


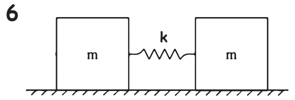
F=-kx
$T=2\pi\sqrt{\frac{m}{k}}$

4


F=-kx
$T=2\pi\sqrt{\frac{m}{k}}$

5


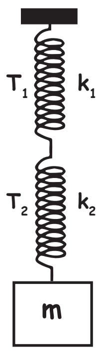
$T=2\pi\sqrt{\frac{m}{k+\rho Hg}}$

6


F=-kx
$T=2\pi\sqrt{\frac{m_{rel}}{k}}$
$m_{rel}=\frac{m_1m_2}{m_1+m_2}$

COMBINATIONS OF SPRINGS

1) Series



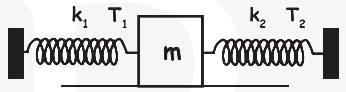
$k_{eq}=\frac{k_1k_2}{k_1+k_2}$

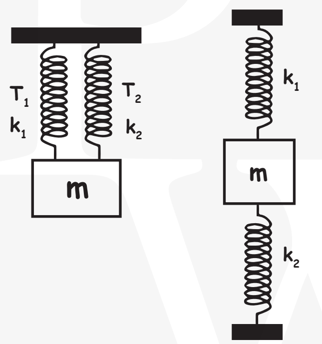
$T=2\pi\sqrt{\frac{m}{k_{eq}}}$

$T_s^2=T_1^2+T_2^2$

$T_s=\sqrt{T_1^2+T_2^2}$

2) Parallel



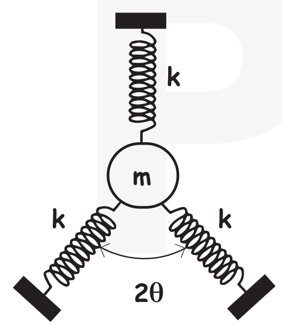


$k_{eq}=k_1+k_2$

$T=2\pi\sqrt{\frac{m}{k_{eq}}}$

$\frac{1}{T_p^2}=\frac{1}{T_1^2}+\frac{1}{T_2^2}$

SPECIAL CASE



$k_{eq}=k+2k\cos^2\theta$

$T=2\pi\sqrt{\frac{m}{k_{eq}}}$

eg:

if $\theta=120°, \theta=60°$

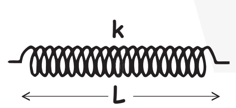
$k_{eq}=k+\frac{2k}{4}=\frac{3k}{2}$

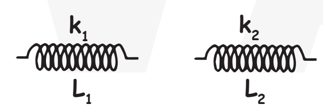
$T=2\pi\sqrt{\frac{2m}{3k}}$

CUTTING OF SPRINGS

$k\propto\frac{1}{L}$

$k_1L_1=k_2L_2$



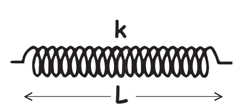


$L=L_1+L_2$

$k_1=\frac{kL}{L_1}$ $\quad k_2=\frac{kL}{L_2}$

$k_1=\frac{k(L_1+L_2)}{L_1}$ $\quad k_2=\frac{k(L_1+L_2)}{L_2}$

eg:



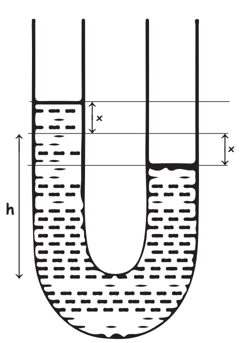
$L_1:L_2:L_3=1:2:3$

$k_1=\frac{k(L_1+L_2+L_3)}{L_1}=\frac{k6}{1}=6k$

$k_2=\frac{k6}{2}=3k$

$k_3=\frac{k6}{3}=2k$

OSCILATION OF LIQUID COLUMN

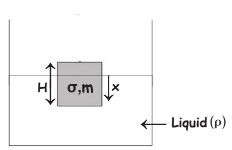


$T=2\pi\sqrt{\frac{m}{2\rho gA}}$

$T=2\pi\sqrt{\frac{h}{g}}$

ρ=Density of liquid
A=Area of U-tube

OSCILATION OF FLOATING BODY

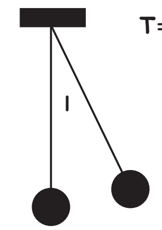


$T=2\pi\sqrt{\frac{m}{\rho gA}}$

$T=2\pi\sqrt{\frac{\sigma H}{\rho g}}$

σ=Density of Body
ρ=Density of liquid

SIMPLE PENDULUM



$T=2\pi\sqrt{\frac{l}{g}}$

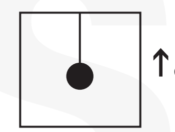
- Second's pendulum
  T=2 second
  l=1 meter

Concept of $g_{effective}$

$T=2\pi\sqrt{\frac{l}{g_{eff}}}$

Pendulum in lift

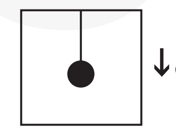
Case 1-Moving up with constant acceleration 'a'



$g_{eff}=g+a$

$T=2\pi\sqrt{\frac{l}{g+a}}$

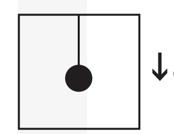
Case 2-Moving down with constant acceleration 'a'



$g_{eff}=g-a$

$T=2\pi\sqrt{\frac{l}{g-a}}$

Case 3-Moving with constant velocity



a=0

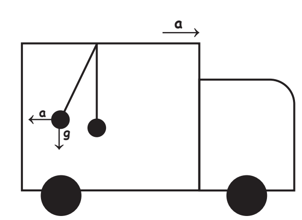
$T=2\pi\sqrt{\frac{l}{g}}$

Case 4-Free fall

a=g

$g_{eff}=g-a=g-g=0 \quad T\to\infty$

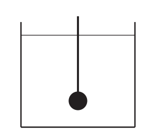
Pendulum in a truck moving with constant acceleration



$g_{eff}=\sqrt{g^2+a^2}$

$T=2\pi\sqrt{\frac{l}{\sqrt{g^2+a^2}}}$

Pendulam in Water



σ=Density of Bob
ρ=Density of liquid

$g_{eff}=g\left(1-\frac{\rho}{\sigma}\right)$

$T=2\pi\sqrt{\frac{l}{g\left(1-\frac{\rho}{\sigma}\right)}}$

DIFFERENTIAL EQUATION OF S.H.M

ma= -kx

$m\frac{d^2x}{dt^2}=-kx$

$\frac{d^2x}{dt^2}=\frac{-k}{m}x$

$\frac{d^2x}{dt^2}+\frac{k}{m}x=0$

$\frac{d^2x}{dt^2}+\omega^2x=0$

$\omega^2=\frac{k}{m} \quad \omega=\sqrt{\frac{k}{m}}$

A=Amplitude of SHM
Φ_0=Initial phase difference

Eg: $\begin{bmatrix} m = 4.\ Kg \\ K = 320\ N/m \end{bmatrix}$

$4\frac{d^2x}{dt^2}+320x=0$

$\frac{d^2x}{dt^2}+80x=0$

$\omega^2=80 \quad \omega=\sqrt{80}$

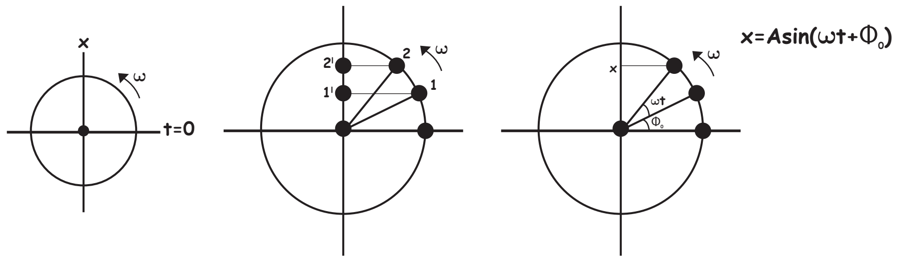
$\frac{2\pi}{T}=\sqrt{80}$

$T=\frac{2\pi}{\sqrt{80}}$

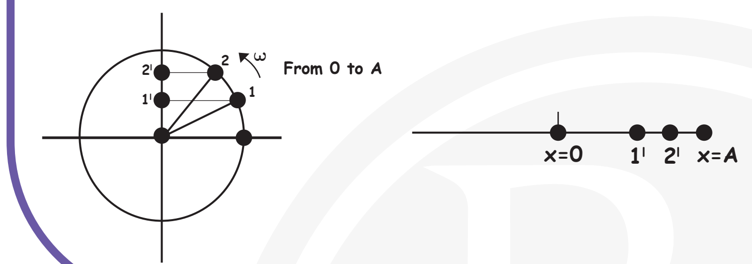
PHYSICS WALLAH

OSCILLATIONS 02

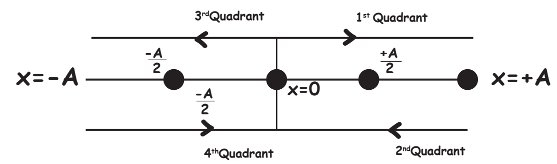
PROJECTION OF CIRCULAR MOTION



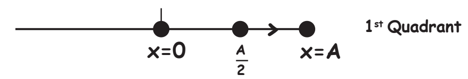
Projection/shadow of uniform circular motion in y axis is SHM



INITIAL PHASE FROM POSITION & DIRECTION



Eg: Particle lies at $x=\frac{A}{2}$ [t=0] and move towards A



1st Quadrant

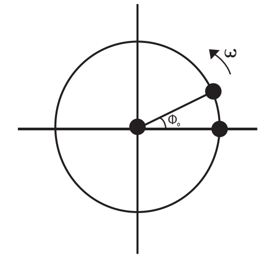
Start from mean position

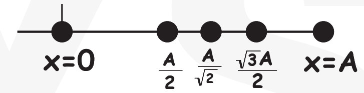
$x=A\sin(\omega t+\Phi_0)$

1) $x=\frac{A}{2}$, t=0 — $\frac{A}{2}=A\sin\Phi_0$, $\Phi_0=30^\circ=\frac{\pi}{6}$, $x=A\sin(\omega t+\frac{\pi}{6})$

2) $x=\frac{A}{\sqrt{2}}$, t=0 — $\frac{A}{\sqrt{2}}=A\sin\Phi_0$, $\Phi_0=45^\circ=\frac{\pi}{4}$, $x=A\sin(\omega t+\frac{\pi}{4})$

3) $x=\frac{\sqrt{3}A}{2}$, t=0 — $\frac{\sqrt{3}A}{2}=A\sin\Phi_0$, $\Phi_0=60^\circ=\frac{\pi}{3}$, $x=A\sin(\omega t+\frac{\pi}{3})$





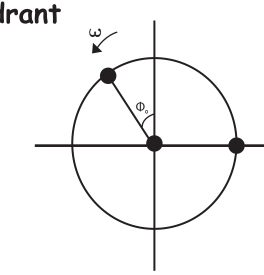
2nd Quadrant

Start from extreme position

$x=A\cos(\omega t+\Phi_0)$

1) $x=\frac{A}{2}$, t=0 — $\frac{A}{2}=A\cos\Phi_0$, $\Phi_0=60^\circ=\frac{\pi}{3}$, $x=A\cos(\omega t+\frac{\pi}{3})$

2) $x=\frac{A}{\sqrt{2}}$, t=0 — $\frac{A}{\sqrt{2}}=A\cos\Phi_0$, $\Phi_0=45^\circ=\frac{\pi}{4}$, $x=A\cos(\omega t+\frac{\pi}{4})$



Velocity of SHM

$x=A\sin(\omega t+\Phi_0)$

$v=A\omega\cos(\omega t+\Phi_0)$

$v=\omega\sqrt{A^2-x^2}$

a) $x=0$ — $v_{max}=A\omega$

b) $x=\frac{A}{2}$ — $v=\frac{\sqrt{3}A\omega}{2}$

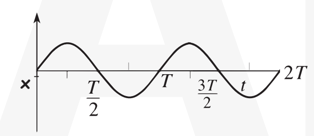
c) $x=\frac{A}{\sqrt{2}}$ — $v=\frac{A\omega}{\sqrt{2}}$

d) $x=A$ — $v=0$

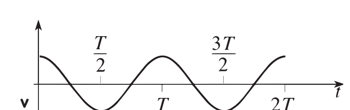
Graphical Representation of Velocity

Start from mean position

$x=A\sin(\omega t+\Phi_0)$ $\Phi_0=0$

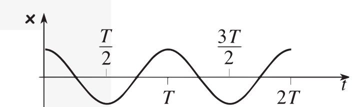


$v=A\omega\cos(\omega t+\Phi_0)$

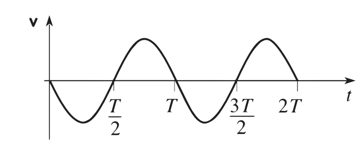


Start from extreme position

$x=A\cos(\omega t+\Phi_0)$ $\Phi_0=0$



$v=-A\omega\sin(\omega t+\Phi_0)$



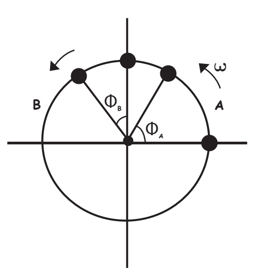
Two particles executing SHM meet at $x=\frac{\sqrt{3}A}{2}$

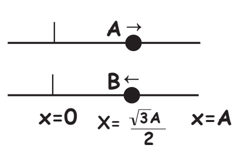
for particle A

$x=\frac{\sqrt{3}A}{2}$, t=0 — $\frac{\sqrt{3}A}{2}=A\sin\Phi_A$, $\Phi_A=60^\circ=\frac{\pi}{3}$

for particle B

$x=\frac{\sqrt{3}A}{2}$, t=0 — $\frac{\sqrt{3}A}{2}=A\cos\Phi_B$, $\Phi_B=30^\circ=\frac{\pi}{6}$





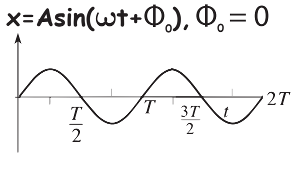
Phase difference

$\Phi=30^\circ+30^\circ=60^\circ$ or 300°

Graphical Representation of Displacement

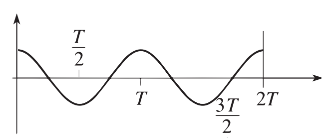
Start from mean position

$x=A\sin(\omega t+\Phi_0)$, $\Phi_0=0$

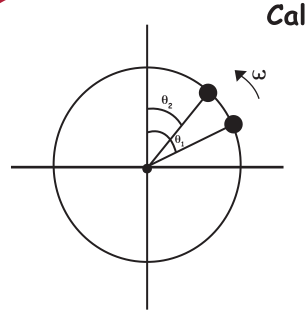


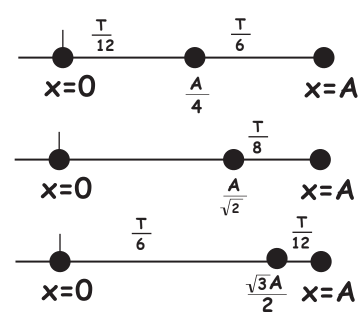
Start from extreme position

$x=A\cos(\omega t+\Phi_0)$, $\Phi_0=0$

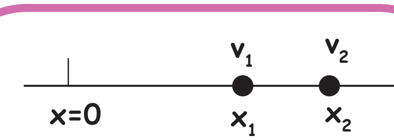


Calculation of time





$\Delta t=\frac{\Delta\theta}{\omega}=\left(\frac{\theta_1-\theta_2}{2\pi}\right)T$



$A=\sqrt{\frac{v_1^2x_2^2-v_2^2x_1^2}{v_1^2-v_2^2}}$ $\omega=\sqrt{\frac{v_1^2-v_2^2}{x_2^2-x_1^2}}$

PHYSICS WALLAH

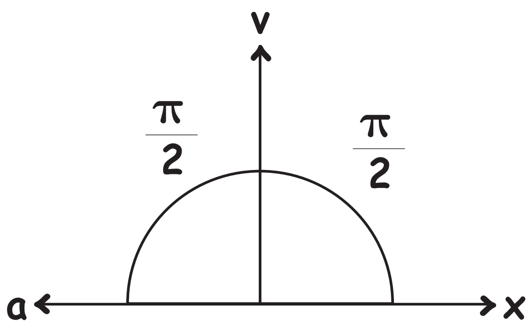
Acceleration of SHM

$x = A\sin(\omega t + \Phi_0)$

$v = A\omega\cos(\omega t + \Phi_0)$

$a = -A\omega^2\sin(\omega t + \Phi_0)$

$a_{max} = -A\omega^2$



Phase difference between x and v= $\frac{\pi}{2}$

Phase difference between v and a= $\frac{\pi}{2}$

Phase difference between x and a= π

Calculation of Time period and amplitude

$v_{max} = A\omega$ $\qquad a_{max} = A\omega^2$

$\omega = \frac{a_{max}}{v_{max}}$ $\qquad A = \frac{v_{max}^2}{a_{max}}$

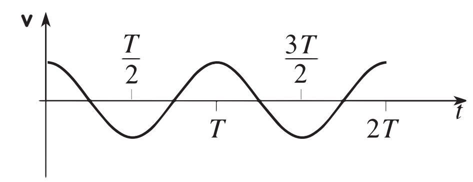
$\frac{2\pi}{T} = \frac{a_{max}}{v_{max}}$

$T = 2\pi\frac{v_{max}}{a_{max}}$

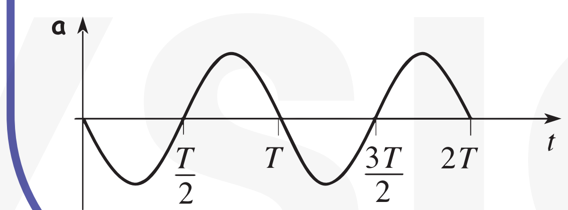
Graphical Representation of Acceleration

Start from mean position

$v = A\omega\cos(\omega t + \Phi_0)$, $\Phi_0 = 0$

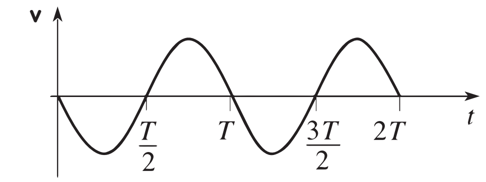


$a = -A\omega^2\sin(\omega t + \Phi_0)$

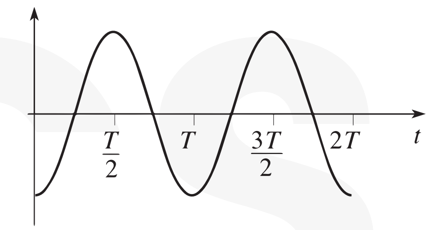


Start from extreme position

$v = -A\omega\sin(\omega t + \Phi_0)$, $\Phi_0 = 0$



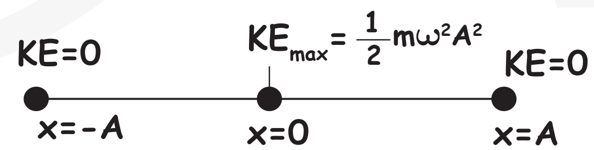
$a = -A\omega^2\cos(\omega t + \Phi_0)$



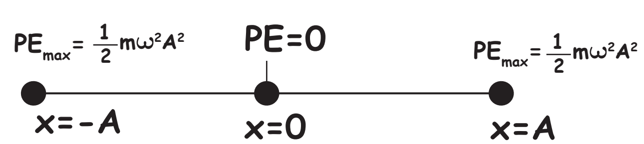
OSCILLATIONS 03

Energy of SHM

$K.E = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2)$



$P.E = \frac{1}{2}m\omega^2x^2$



Total mechanical energy $= \frac{1}{2}m\omega^2A^2$

1) $x = \frac{A}{\sqrt{2}}$

$K.E = P.E = \frac{E}{2}$

2) $x = \frac{A}{2}$

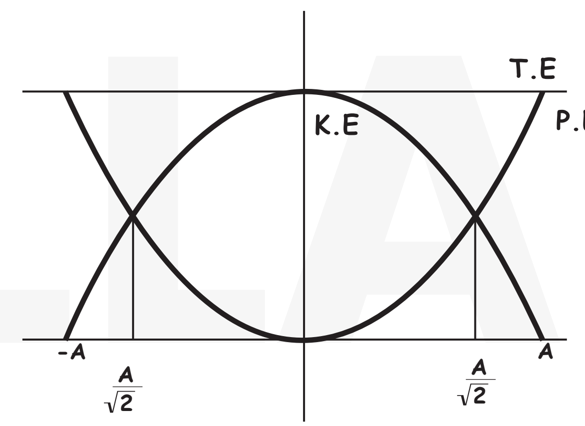
$K.E = \frac{3E}{4}$ $\qquad P.E = \frac{E}{4}$

3) $x = 0$

$K.E = E$ $\qquad P.E = 0$

4) $x = A$

$K.E = 0$ $\qquad P.E = E$



Total mechanical energy(E) $= \frac{1}{2}m\omega^2A^2$

Note: In SHM, if particle oscilate with frequency ω, then the K.E & P.E oscilate with 2ω